Arithmetic Progressions - Almost Periodicity

Yaël Dillies, Sorrachai Yingchareonthawornchai

October 8, 2026

Chapter 1

Definitions

Definition 1.1 (Matrix containment)**.** *An n-by-m binary matrix M contains an n'-by-m' matrix P if there are integers $1 \leq i_1 \leq \ldots \leq i_{n'} \leq n$ and $1 \leq j_1 \leq \ldots \leq j_{m'} \leq m$ such that for all $i' \in [n'], j' \in [m']$, $P(i', j') = 1$ implies $M(i_{i'}, j_{j'}) = 1$. If M does not contain P, then M avoids P.*

Definition 1.2 (Extremal Function)**.** *Given two integers n, m, we denote $Ex(P, n, m)$ as the maximum number of 1's in a matrix avoiding a matrix P. We also define $Ex(P, n) = Ex(P, n, n)$.*

Chapter 2

The two points matrix

Chapter 3

The tower matrix

Chapter 4

The tetris matrix

Chapter 5

Permutation matrices